EQUILIBRIUM DISCHARGE FROM A NOZZLE OF A MIXTURE OF AN IDEAL GAS AND A CONDENSING VAPOR

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Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 2, pp. 188–196, 1966

UDC 533.6

Within the framework of the theory of ideal gases, an examination is made of equilibrium discharge of a mixture of a gas and a condensing vapor from a nozzle. The basic equations describing discharge are presented, as well as the results of calculations of the discharge from a nozzle of a mixture of air and water vapor.

Depending on the expansion rate and the degree of impurity due to mechanical mixing (dust particles, etc.), the discharge of a mixture of a gas and a saturated vapor of any liquid may have either a nonequilibrium (with condensation shocks) [1] or an equilibrium character. We shall examine the equilibrium discharge of such a mixture from a nozzle, making a number of assumptions to permit the problem to be appreciably simplified.

We assume the following conditions: a) the flow is steady and one-dimensional, and there is no heat exchange with the surrounding medium; b) the expansion process is an equilibrium one, and the mixture is completely homogeneous; c) the expansion takes place in regions far enough from critical points for both the gas and the dry saturated vapor to be considered as ideal gases; d) at the start of the expansion of the mixture, the vapor is in the dry saturated state; e) the specific heat of the gas is constant; f) the velocity of particles formed in equilibrium condensation of the vapor is equal to that of the flow itself (i.e., the flow is also considered to be in equilibrium with respect to velocity).

The last supposition requires some clarification. The author's calculations show that the velocities of particles formed during equilibrium condensation of a moist vapor are very close to the velocity of the vapor. This is explained by the fact that the initial size of the particles is very small, and so they are easily entrained by the vapor. In addition, the vapor molecules condensing on the surface of a particle have a mean velocity equal to that of the vapor stream at any section; this also leads to increase of particle velocity.

Calculation also shows that because of the small difference between the velocities of the particles and the vapor, the losses in friction of the vapor against the particles are negligibly small. Therefore, neglecting also, for simplicity, the usual losses in friction of the vapor against the nozzle walls, the expansion process may be considered isentropic.

As is known from the theory of mixing of ideal gases, the density of the components of a mixture is determined from the partial pressures and the temperature. Then the temperatures (and also the velocities) of all the components are considered identical, because of the assumed homogeneity of the mixture. We may therefore write an independent mass flow equation for each component of the mixture, and for the mixture of gas and dry saturated vapor examined here the equations have the form:

$$G_g = G_{g0} = F\rho_g c,$$
$$G_v = G_{v0}x = F\rho_v c.$$

Referring the first equation to 1 kg of gas, it may be put in the form

$$1 = f\frac{\rho_g}{\rho_{g0}}\frac{\bar{c}}{\tau}, \tag{1}$$

where

$$f = F/F_0, \quad \bar{c} = c/c_0, \quad \tau = T/T_0.$$

Dividing the second equation by the first, we obtain the following ratio between the concentrations of gas and vapor:

$$\frac{g_v}{g_g} = g_{v0}x/g_{g0} = \frac{\rho_v}{\rho_g}. \tag{2}$$

Under the assumption of equal velocities of gas, vapor, and condensate particles, the momentum equation of the mixture may be written in the form

$$-\frac{dp_m}{\rho_m} = -g_g\frac{dp_g}{\rho_g} - g_v\frac{dp_v}{\rho_v} = \left(1 + g_v\frac{1-x}{x}\right)cdc.$$

The second term on the right of this equation is due to the presence, in 1 kg of gaseous components of the mixture, of $g_v(1 - x)/x$ kg of condensate particles. This equation, referred to 1 kg of working substance (i.e., mixture of gas, dry saturated vapor, and condensate particles), may be written in the following integral form:

$$-g_{g0}\int\frac{dp_g}{\rho_g} - g_{v0}\int\frac{dp_v}{\rho_v}x = \frac{c^2 - c_0^2}{2}. \tag{3}$$

The left side of this equation is the sum of partial work done in expanding g_{g0} kg of gas and g_{v0} kg of moist vapor, respectively.

Using the energy equation and some other relations to determine the form of the functions ρ_g, ρ_v, and x, we may calculate values of the integrals which appear in the expressions for partial work of the gas and

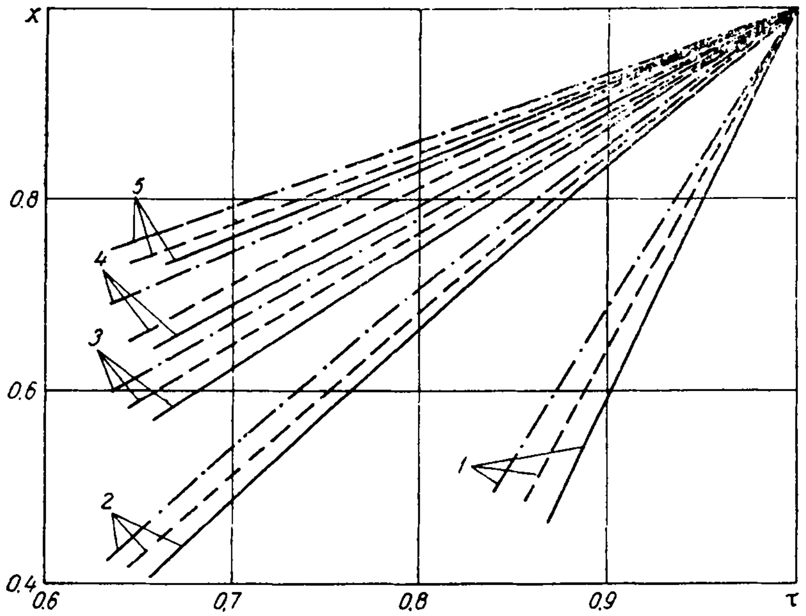

Fig. 1. Dependence of the degree of dryness, x, of the vapor component of the mixture on dimensionless mixture temperature $\tau = T/T_0$, when m = 4, 3.5, and 3 (continuous, broken, and dot-dash lines, respectively) and g_V = 0.1 (1), 0.5 (2), 1 (3), 2 (4), and ∞ (5).

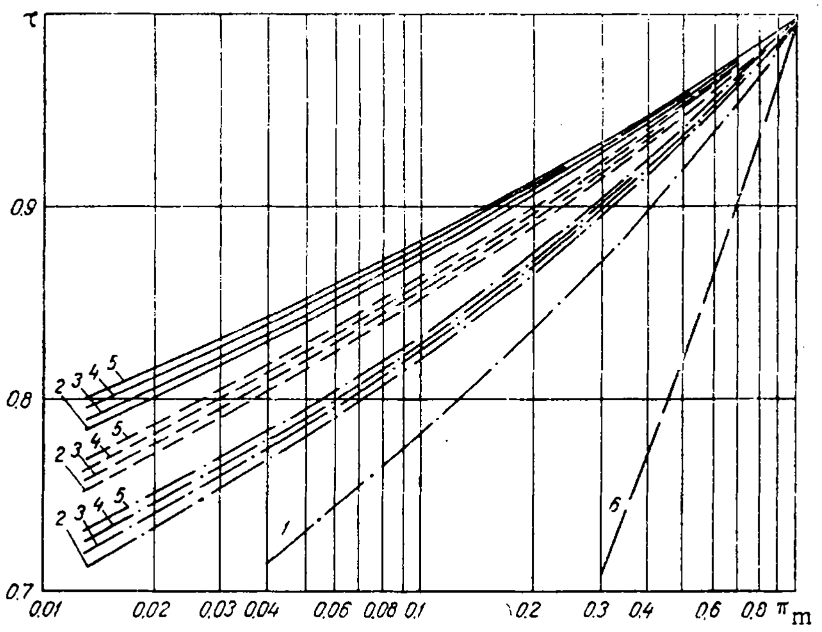

Fig. 2. Dependence of dimensionless mixture temperature τ on the mixture pressure drop $\pi_m = p_m/p_{m0}$ when m = 4, 3.5, and 3 (continuous, broken, and dot-dash lines, respectively) and g_V = 0.1 (1), 0.5 (2), 1 (3), 2 (4), ∞ (5); curve 6 refers to pure air (g_V = 0).

moist vapor. We shall show, however, that a determination of the total work in expanding the mixture may be accomplished in another, simpler manner, and to do this we shall examine in more detail the process of expanding a gaseous and vapor phase.

When a mixture is expanded the vapor condenses. Part of the heat of condensation then liberated is supplied to the gas and increases its work capacity somewhat in comparison with the case of adiabatic expansion. Therefore the work of expanding 1 kg of gas in this case may be represented in the form

$$H_g = -\int \frac{dp_g}{\rho_g} = H_{g.ad} + \Delta A_Q ,$$

where $H_{g \cdot ad} = c_{pg} T_0 [1 - (p_g/p_{g0})^{(k-1)/k}]$ is the work in adiabatic expansion of 1 kg of gas, and ΔA_Q is the increase in work capacity of 1 kg of gas due to the supply of heat in the expansion process.

When the mixture cools by dT during expansion, $(\sim g_{v0}/g_{g0})dx$ kg of gas is condensed and $r(g_{v0}/g_{g0})dx$ of heat is liberated; in addition, $(g_{v0}/g_{g0})(1 - x)c_{pl}dT$ of heat is liberated in cooling by $dT(g_{v0}/g_{g0})(1 - x)$ kg of condensate. The heat liberated is, altogether,

$$dQ = -r \frac{g_{g0}}{g_{g0}} dx - \frac{g_{v0}}{g_{g0}} (1 - x) c_{pl} \, dT .$$

Part of the heat released, however, goes to heating the vapor phase, which remains throughout in the dry saturated state. As is known [2], the specific heat of a vapor, when its state is changed along the right branch of the boundary curve, is equal to

$$c_v = c_{pl} + \frac{dr}{dT} - \frac{r}{T} .$$

Therefore, the amount of heat that goes to heat $(g_{v0}/$ $/g_{g0})x$ kg of vapor is

$$dQ_v \quad \frac{g_{v0}}{g_{g0}} x c_v dT = \frac{g_{v0}}{g_{g0}} x dr + \frac{g_{v0}}{g_{g0}} \left(c_{pl} - \frac{r}{T} \right) x dT .$$

Thus, the heat supplied to 1 kg of gas is

$$dQ_g = dQ - dQ_v =$$

$$= -\frac{g_{v0}}{g_{g0}} \left[d(rx) + c_{pl} \, dT - \frac{r}{T} x dT \right] . \qquad (4)$$

This amount of heat increases the work capacity of the gas, in comparison with the adiabatic process, by an amount

$$dA_Q = (1 - T_1/T) dQ_g .$$

The total increase in work capacity of the gas due to supply of condensation heat is

$$\Delta A_Q = \int_{T_0}^{T} dQ_g - T \int_{T_0}^{T} \frac{dQ_g}{T} .$$

Substituting the expression for dQ_g and carrying out the integration, we obtain

$$\Delta A_Q =$$

$$- \frac{g_{v0}}{g_{g0}} \left[\left(1 - \frac{T}{T_0} \right) (r_0 + c_{pl} T_0) + c_{pl} T \ln \frac{T}{T_0} + \int_{T_0}^{T} \frac{rx}{T} dT \right] .$$

We also transform to some extent the expression for the work of expansion of the vapor, appearing on the left side of (3). From the Clausius-Clapeyron equation [2]

$$\frac{dp_v}{dT} = \frac{r}{T (V_v - V_l)} , \qquad (5)$$

neglecting the specific volume of condensate V_l in comparison with the specific volume of the dry saturated vapor, we easily obtain

$$\frac{dp_v}{\rho_v} = \frac{r}{T} dT .$$

Thus, the work in expanding 1 kg of moist vapor is

$$H_v = -\int_{p_{v0}}^{p_v} x \frac{dp_v}{\rho_v} = -\int_{T_0}^{T} \frac{rx}{T} dT .$$

Using the relations obtained for the work of the gaseous and vapor phases, we may put the left side of (3) in the form

$$H_m = g_{g0} c_{pg} T_0 [1 - (p_g/p_{g0})^{(k-1)/k}] +$$

$$+ g_{v0} [(1 - T/T_0)(r_0 + c_{pl} T_0) + c_{pl} T \ln (T/T_0)] . \quad (6)$$

The second term on the left of this equation is known to be the work in expanding g_{v0} kg of moist vapor adiabatically. We thus obtain the result that the total work of expansion of 1 kg of mixture is equal to the sum of the available work of g_{g0} kg of gas in an adiabatic expansion from the initial to the final partial pressures, and of the available work of g_{v0} kg of vapor in an adiabatic expansion from the initial state to the final temperature of the mixture.

Taking into account the last relation and assuming r to be a linear function of temperature ($r = a - bT$), the momentum equation of the mixture (3) may be put in the following dimensionless form:

$$1 - (p_g/p_{g0})^{(k-1)/k} + \bar{g}_v \frac{b}{c_{pl}} (m - 1) \frac{c_{pl}}{c_{pg}} \cdot \qquad (3')$$

$$\times \left[(1 - \tau) \left(1 + \frac{c_{pl}}{b} \frac{1}{m - 1} \right) + \frac{c_{pl}}{b} \frac{\tau \ln \tau}{m - 1} \right] = \frac{\delta (\bar{c}^2 - 1)}{g_{g0}} ,$$

where

$$\bar{g}_v = g_{v0}/g_{g0}, \quad m = a/bT_0, \quad \delta = c_0^2/2c_{pg}T_0 .$$

We shall now derive the equations of the processes of expansion of the gaseous and vapor phases. When a

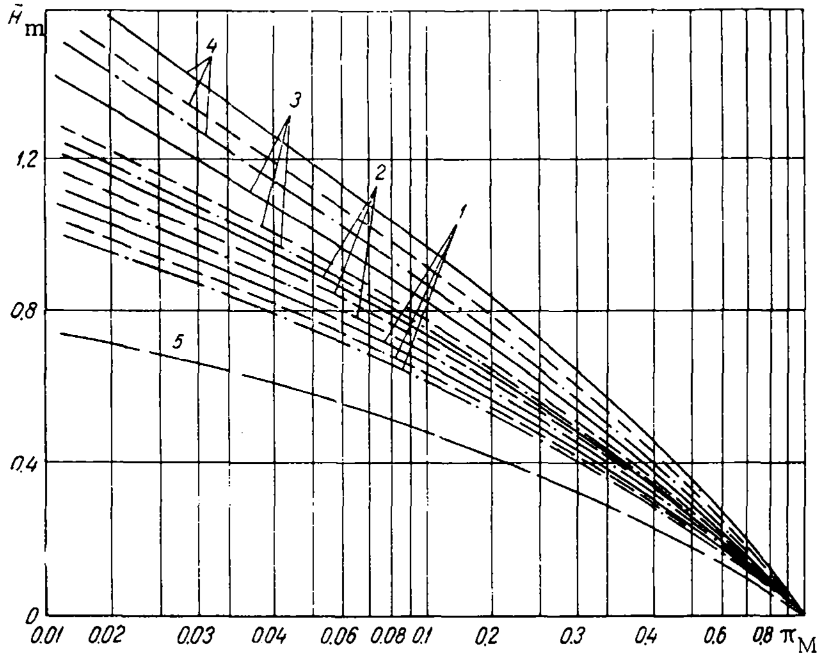

Fig. 3. Dependence of the dimensionless available energy $\overline{H}_m = H_m/c_{pg}T_0$ of the mixture on the mixture pressure drop π_m, with m = 4, 3.5, and 3 (continuous, broken, and dot-dash lines, respectively) and $g_v = 0.5$ (1), 1 (2), 2 (3), ∞ (4); curve 5 refers to pure air ($g_v = 0$).

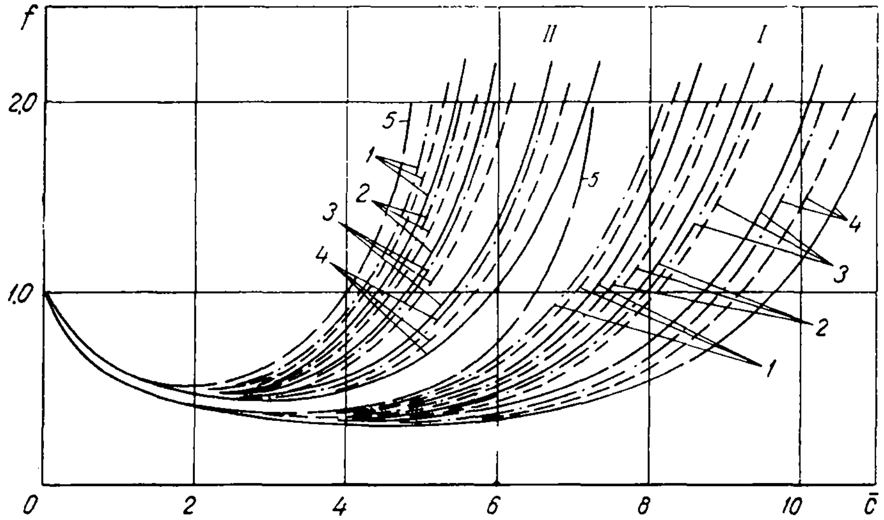

Fig. 4. Dependence of relative nozzle area $f = F/F_0$ on dimensionless flow velocity $\bar{c} = c/c_0$, with $\delta = 0.01$ (I) and 0.02 (II); m = 4, 3.5, and 3 (continuous, broken, and dot-dash lines, respectively) and $g_v = 0.5$ (1), 1 (2), 2 (3), ∞ (4), and 0 (5).

certain amount of heat of entropy is supplied to the gas during expansion, its S increases. At the same time, the temperature T_1 and the enthalpy i_{g1} of the gas at the final pressure p_1 increase. When an amount of heat dQ_g is supplied, the enthalpy increases by

$$di_{g1} = c_{pg}dT_1 = T_1 dS = T_1 \frac{dQ_g}{T}.$$

Hence

$$\frac{dT_1}{T_1} = \frac{dQ_g}{Tc_{pg}},$$

where dQ_g is determined from (4).

Integrating this equation, and bearing in mind that when $T = T_0$, $T_1 = T_{ad}$, $x = 1$, $r = r_0$, at the end of the expansion $T = T_1$, and that $T_0/T_{ad} = (p_{g0}/p_{g1})^{(k-1)/k}$ (where T_{ad} is the final temperature in an adiabatic expansion of the gas), we finally obtain the following relation between the partial pressure of the gas and its temperature during expansion in the mixture:

$$\frac{p_g}{p_{g0}} = \tau^{\frac{k}{k-1}\left(\bar{g}_v \frac{c_{pl}}{c_{pg}}+1\right)} \exp\left[\frac{k}{k-1} \bar{g}_v \frac{c_{pl}}{c_{pg}} \times \frac{b}{c_{pl}}(m-1)\left(\frac{m-\tau}{m-1}\frac{x}{\tau}-1\right)\right]. \qquad (7)$$

We shall find an analogous relation between the pressure and the temperature for the vapor phase. From the Clausius-Clapeyron equation (5), using the equation of state for ideal gases, we obtain

$$\frac{dp_v}{p_v} = \frac{rdT}{R_v T^2}.$$

In the case $r = a - bT$, this equation may easily be integrated, the solution having the form

$$\frac{p_v}{p_{v0}} = \tau^{-b/R_v} \exp\left[m\frac{b}{R_v}\left(1-\frac{1}{\tau}\right)\right]. \qquad (8)$$

Dividing (8) by (7) and taking account of (2), we obtain the following expression for determining the degree of dryness x of the vapor as a function of the dimensionless temperature τ of the mixture:

$$x = \exp\left[\left(\frac{b}{R_v} + \frac{k}{k-1}\bar{g}_v\frac{c_{pl}}{c_{pg}} + \frac{k}{k-1}\right)\ln\frac{1}{\tau}\right] \times \left\{\exp\left[m\frac{b}{R_v}\left(\frac{1}{\tau}-1\right) + \frac{k}{k-1}\bar{g}_v(m-1)\frac{b}{c_{pl}} \cdot \frac{c_{pl}}{c_{pg}}\left(\frac{m-\tau}{m-1}\frac{x}{\tau}-1\right)\right]\right\}^{-1}.$$

This equation contains six characteristic parameters:

$$k,\ b/R_v,\ c_{pl}/c_{pg},\ b/c_{pl},\ \bar{g}_v,\ m.$$

Of these, the first four depend only on the physical constants of the gas or vapor; therefore discharge of a mixture of given components is fully determined by the remaining two parameters:

$$\bar{g}_v = g_{v0}/g_{g0} \text{ and } m = a/bT_0.$$

From (9) it is easy to obtain values of the degree of dryness of the vapor when its content in the mixture is very small or very large. In fact, let $\bar{g}_v \to 0$ (i.e., the gas has insignificant vapor impurity). In this case

$$x_0 = \lim_{\bar{g}_v \to 0} x = \left\{\exp\left[\ln\frac{1}{\tau}\left(\frac{b}{R_v}+\frac{k}{k-1}\right)\right]\right\} \times \left\{\exp\left[m\frac{b}{R_v}\left(\frac{1}{\tau}-1\right)\right]\right\}^{-1}.$$

In the case $\bar{g}_v \to \infty$ (flow of a condensing vapor with insignificant gas impurity), Eq. (9) may be brought to the following form:

$$x = x_0 \exp\left\{\frac{k}{k-1}\frac{c_{pl}}{c_{pg}}\bar{g}_v\left[\ln\frac{1}{\tau}-(m-1)\frac{b}{c_{pl}} \times \left(\frac{m-\tau}{m-1}\frac{x}{\tau}-1\right)\right]\right\}.$$

Since the degree of dryness of the vapor is $0 \le x \le 1$, i.e., its value is considerably limited, we must have, when $\bar{g}_v \to \infty$,

$$\ln\frac{1}{\tau} - (m-1)\frac{b}{c_{pl}}\left(\frac{m-\tau}{m-1}\frac{x}{\tau}-1\right) \to 0.$$

From this expression we easily find the limiting value of x when $\bar{g}_v \to \infty$:

$$x_\infty = \lim_{\bar{g}_v \to \infty} x = \frac{m-1}{m-\tau}\tau - \frac{c_{pl}}{b}\frac{\tau\ln\tau}{m-\tau}.$$

The expression obtained agrees with the usual equation for the dryness of a vapor adiabatically expanded from the dry saturated state.

Investigation of discharge of a mixture of air and water vapor. We shall examine equilibrium discharge from a nozzle of a mixture of air and water vapor. In this case we may assume [2]: $a = 3200$ kJ/kg, $b = 2.52$ kJ/kg · °K; $r = 3200 - 2.52\ T$; $R_v = 0.461$ kJ/kg · °K; $R_g = 0.288$ kJ/kg · °K; $c_{pl} = 4.2$ kJ/kg × °K; $c_{pg} = 1.01$ kJ/kg · °K; therefore the first four parameters of (9) will have the following values:

$$k = 1.4;\ b/R_v = 5.45;\ c_{pl}/c_{pg} = 4.16;\ b/c_{pl} = 0.6.$$

Equation (9) is solved by trial and error. The results of solving it with the above values of the fixed parameters and various values of the quantities $\bar{g}_v$ and m are shown in Fig. 1. Figure 1 also shows the limiting values of x when a pure vapor is expanded ($\bar{g}_v \to \infty$).

As may be seen by comparing the curves, the dryness resulting from expansion of the vapor in the mixture is appreciably less than in an adiabatic expansion of the pure vapor; the difference increases rapidly as the air content of the mixture is increased. Thus, condensation of the vapor in the air stream is much more intense than in an adiabatic expansion of moist vapor. This is due to the fact that the gas phase strongly absorbs the heat emitted in condensation and plays the role of a cooler for the vapor phase.

It should be noted that in the case of a very small adiabatic exponent the reverse picture may be observed, where the vapor phase is dried during expansion of the mixture, because of the additional supply of heat from the gas.

The quantity π_m in Fig. 2 is determined from the formula

$$\pi_m = \frac{p_g + p_v}{p_{g0} + p_{v0}} = \left(\frac{p_g}{p_{g0}} + \frac{p_v}{p_{v0}} \; \frac{R_v}{R_g} \bar{g}_v \right) \times$$
$$\times \left(1 + \frac{R_v}{R_g} \bar{g}_v \right)^{-1},$$

where p_v/p_{v0} and $p_g/p_{g0} = p_v/p_{v0}x$ are found from (8) and (7), using the value of x determined above.

It may be seen from Fig. 1 that the mixture temperature drops more slowly with decrease of π_m than does the temperature of pure air (lower curve). When the vapor concentration is about 30% and over, the mixture temperature during expansion approximates to the temperature of the pure vapor ($\bar{g}_v \to \infty$).

The quantity $\overline{H}_m$ in Fig. 3 was calculated according to (6), transformed to the form

$$\overline{H}_m = g_{g0} \left[1 - \left(\frac{p_g}{p_{g0}} \right)^{(k-1)/k} \right] + g_{v0} \frac{c_{pl}}{c_{pg}} \times$$
$$\times \left[(m-1) \frac{b}{c_{pl}} + 1 \right] \left[1 - \tau + \right.$$
$$\left. + \tau \ln \tau \Big/ \left(1 + (m-1) \frac{b}{c_{pl}} \right) \right]. \qquad (6')$$

As the water vapor concentration in the mixture is increased, the available energy of 1 kg of mixture, for the same degree of pressure drop, is considerably increased, and approximates to the available energy of the water vapor. The upper curves in Fig. 3 refer to pure water vapor ($\bar{g}_v \to \infty$).

With the aid of the quantity $\overline{H}_m$ it is easy to determine the dimensionless stream velocity and the relative area of the nozzle, from the following formulas, obtained from (3') and (1):

$$\bar{c} = \sqrt{1 + \overline{H}_m/\delta}; \; f = \frac{\tau}{\bar{c}} \; \frac{p_{g0}}{p_g}.$$

It may be seen from the formulas that the quantities $\bar{c}$ and f also depend on the parameter $\delta = c_0^2/2c_{pg}T_0$, which characterizes the initial kinetic energy of the flow.

It may be seen from Fig. 4 that both the concentration and the properties of the vapor have an appreciable influence on the flow-passage cross section of the expanding part of the nozzle. For a single value of relative velocity, the required nozzle area proves to be least for pure vapor. The influence of these parameters is insignificant in the convergent part of the nozzle. We note also that as the vapor concentration in the mixture increases, the throat section of the nozzle shifts in the direction of greater velocities.

NOTATION

G—phase mass flow rate; T, T_1—temperature, and temperature of lowest heat source; p—partial pressure; ρ—density; c—velocity of sound; R—gas constant; k—adiabatic exponent of the gas phase; r—heat of vaporization; c_p—specific heat at constant pressure; F—cross-sectional area of nozzle; x—degree of dryness of vapor; $g_g = G_g/(G_g + G_v)$ and $g_v = G_v/(G_g + G_v)$—stream mass concentrations of gaseous and vapor phases. Subscripts: g—gas; v—vapor; l—liquid; m—mixture; 0—state at the beginning of expansion.

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18 May 1965

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